

On the energy dissipation in confined concrete subjected to shear cyclic loading

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The establishment of a simple engineering rule for predicting the fatigue failure of concrete has been pursued over the past decades. An energetic approach to the matter seems to be an attractive option that many researchers have embraced. In the present work, the authors attempt to contribute to the establishment of such a rule. In particular, the energy dissipation of confined concrete subjected to shear cyclic loading is studied and quantified. For this purpose, a microplane fatigue model recently introduced by the authors, referred to as MS1, is used. It aims to capture the fundamental inelastic mechanisms driving the tri-axial stress redistribution within a material zone during the fatigue damage process in concrete. To this end, the fatigue damage evolution is linked to a measure of cumulative inelastic shear strain at the microplane level, reflecting the accumulation of fatigue damage due to internal shear/sliding between aggregates at subcritical pulsating load levels. To isolate the dissipative mechanism mentioned above, test configurations with dominant shear stress seem to be more appropriate. In the present work, a punch-through shear test (PTST) FE model is used to induce shear-dominated stresses and strains along the ligament of a specimen. Numerical studies are first presented to evaluate the behavior and energy dissipation at the elemental interface level. The interface is introduced in the MS1 microplane material model, which is capable of reproducing the concrete behavior under monotonic, cyclic, and fatigue loading with consistent set of material parameters. Quantification of the energy dissipation for each introduced dissipative mechanism is performed at each microplane and integrated via a well-established homogenization scheme to evaluate the macroscopic energy dissipation. Later, an analysis of the energy dissipation of the PTST process zone is performed for cyclic loading under two different subcritical cyclic load amplitudes.

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1 Introduction

Accurate prediction of fatigue life of concrete structures remains an unsolved issue that increasingly attracts the attention of the research community [1–4]. Deeper understanding of concrete fatigue is urgently needed to address the ecological and sociological challenge of the next decade, namely the trade-off between the high carbon footprint associated with the concrete construction sector on the one hand, and a world-wide increasing demand for transportation and energy infrastructure made of concrete on the other hand. To meet this challenge, new class of eco-efficient binders must be introduced in the engineering practice. At the same time, lifetime of structures must be extended. Such a radical transition can only be achieved based on a general and rigorous description of fatigue phenomenology both at the material and at the structural levels.

The hypothesis proposed recently by the authors postulates that fatigue damage evolution in concrete is primarily related to the inter-aggregate cumulative inelastic shear strain, i.e. to the cumulative relative shear strain or slip displacements between aggregates occurring at pulsating subcritical load levels. By making use of this hypothesis the fatigue behavior of cementitious interfaces [5, 6] and concrete under compressive fatigue loading [7] could be realistically modeled. To validate the hypothesis in a broad range of fatigue stress configurations, the cylindrical punch through shear test (PTST) configuration has been recognized to provide more flexibility regarding the fatigue stress configurations, thanks to the the possibility of independently varying the level of combined confinement and cyclic shear loading in a tested material zone.

2 Microplane model for fatigue MS1

To evaluate the potential of PTST, we use the recently developed material model MS1 [7], introducing the cumulative shear fatigue hypothesis at the microplane level, in an axi-symmetric idealization of the test specimen. By linking fatigue evolution to a cumulative measure of inelastic shear strain the tri-axial stress redistribution within the material structure in response to pulsating subcritical loading can be reproduced. The fatigue-induced stress redistribution corresponds to the experimentally observed degradation process presented e.g. by Skarzynski et al. [8], indicating that crack initiation and propagation during fatigue loading occurs along interfaces between the hardened cement paste and aggregates. The model, formulated within the microplane framework, makes use of the kinematic constraint in order to project the macroscopic strain tensor onto the microplanes and the principle of virtual work and energy equivalence in order to homogenize the microplane stresses, secant and plastic strain tensors.

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2.1 Thermodynamically based formulation

Thermodynamically based constitutive laws capturing the degradation mechanisms are introduced at the level of the idealized microplane, an oriented plane with an ascribed dissipative behavior that represents the inter-aggregate behavior within a 3D material structure in the corresponding direction. The macroscopic Helmholtz free energy is expressed as the sum of the microplane normal and tangential thermodynamic potentials:

$$\psi^{\text{mac}} = \frac{3}{2\pi} \int_{\Omega} \psi^{\text{mic}} d\Omega = \frac{3}{2\pi} \int_{\Omega} \psi_N d\Omega + \frac{3}{2\pi} \int_{\Omega} \psi_T d\Omega. \quad (1)$$

The projection of the thermodynamic potentials onto the normal and tangential directions allows for the distinction of individual dissipative mechanisms for each direction, as summarized in Fig. 2 c.

Normal direction: The microplane thermodynamic potential for the normal direction is defined as

$$\rho\psi_N^{\text{mic}} = \frac{1}{2} [1 - H(\sigma_N)\omega_N] E_N (\varepsilon_N - \varepsilon_N^p)^2 + \frac{1}{2} K_N z_N^2 + \frac{1}{2} \gamma_N \alpha_N^2, \quad (2)$$

where ρ denotes the material density and the normal elastic stiffness $E_N = E/(1 - 2\nu)$, E the elastic modulus and ν the Poisson's ratio. The Heaviside function $H(\sigma_N)$ enables the switch of normal behavior between tension and compression. If the microplane is subjected to compression in the normal direction, $H(\sigma_N < 0) = 0$, and plastic process can develop while normal damage remains unchanged. In case of tension $H(\sigma_N > 0) = 1$, the damage process can take place while the normal plastic deformation does not evolve. K_N and γ_N are the isotropic and the kinematic hardening moduli, respectively. The thermodynamic internal variables include the plastic normal strain ε_N^p , the normal damage ω_N , the isotropic and kinematic hardening z_N, α_N . The conjugate thermodynamic forces are determined by differentiating the thermodynamic potential Eq. (2) with respect to each internal variable.

Tangential direction: Cumulative sliding in the tangential direction is considered as the main source of fatigue damage that drives the material deterioration at pulsating subcritical stress levels. To introduce this mechanism, the tangential constitutive behavior of a microplane is introduced similarly to the pressure-sensitive interface model presented earlier in [5], with fatigue damage linked to the cumulative inelastic slip. In analogy to this model, the Helmholtz free energy function of a microplane in the tangential direction is given as

$$\rho\psi_T^{\text{mic}} = \frac{1}{2} (1 - \omega_T) E_T (\varepsilon_T - \varepsilon_T^p) \cdot (\varepsilon_T - \varepsilon_T^p) + \frac{1}{2} K_T z_T^2 + \frac{1}{2} \gamma_T \alpha_T \cdot \alpha_T, \quad (3)$$

where the tangential elastic stiffness $E_T = E(1 - 4\nu)/[(1 + \nu)(1 - 2\nu)]$, and K_T and γ_T are the isotropic and kinematic strain hardening moduli. The thermodynamic internal variables are the inelastic tangential strain vector, ε_T^p , the damage tangential variable ω_T , the isotropic hardening variable z_T and the kinematic hardening vector α_T . As for the normal direction, the conjugate thermodynamic forces are obtained by differentiating the thermodynamic potential (3) with respect to each state variable. The detailed formulation of the model, including the governing threshold functions and the evolution laws can be found in [7].

2.2 Evaluation of the energy dissipation for microplane material model MS1

The evaluation of the energy dissipation is expressed for an isothermal process under small strain assumption by making use of the Clausius-Duhem inequality given for the normal direction as:

$$\dot{D}_{\text{int}}^N = -\rho\dot{\psi}_N + \sigma_N \dot{\varepsilon}_N \geq 0, \quad (4)$$

after substitution for $\dot{\psi}_N$ and application of the chain rule, reads

$$\dot{D}_{\text{int}}^N = - \left(\frac{\partial \rho\psi_N}{\partial \varepsilon_N^e} \dot{\varepsilon}_N - \frac{\partial \rho\psi_N}{\partial \varepsilon_N^p} \dot{\varepsilon}_N^p + \frac{\partial \rho\psi_N}{\partial z_N} \dot{z}_N + \frac{\partial \rho\psi_N}{\partial \alpha_N} \dot{\alpha}_N + \frac{\partial \rho\psi_N}{\partial \omega_N} \dot{\omega}_N \right) + \sigma_N \dot{\varepsilon}_N. \quad (5)$$

Realizing that the derivative of the Helmholtz potential with respect to elastic strain represents the stress variable, i.e.

$$\sigma_N = \frac{\partial \psi_N}{\partial \varepsilon_N^e}, \quad (6)$$

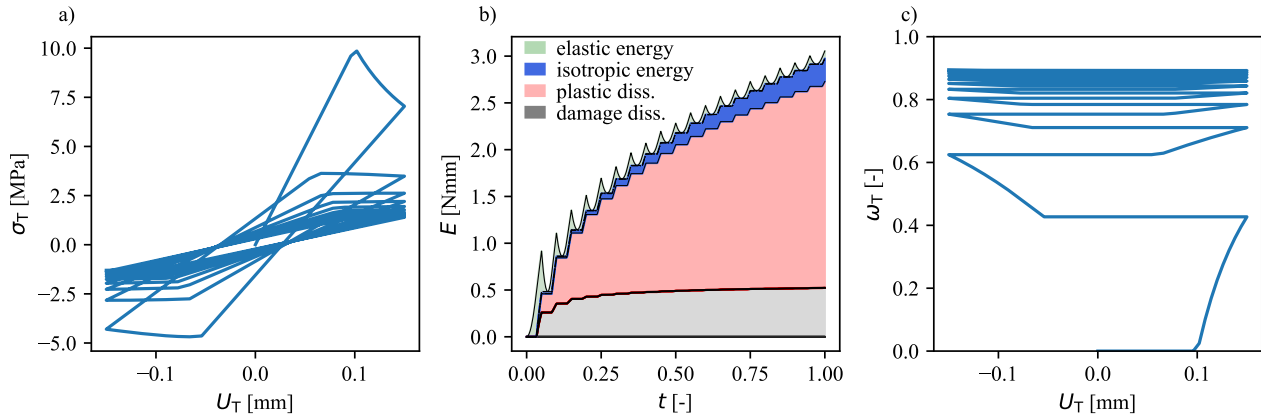


Fig. 1: Elementary interface behaviour. Left figure corresponds to the stress slip curve, middle to the energy breakdown, showing the corresponding fractions of energy dissipation due to damage and plasticity and kinematic and stored energy. Right figure corresponds to the damage evolution with respect to the oscillating displacement loading. Material parameters: $E_T = 100$, $\sigma_T^0 = 10.0$, $\gamma_T = 100$, $K_T = 10$, $S_T = 0.02$, $r_T = 1.0$, $c_T = 2.0$, $p_T = 1.0$, $m_T = 0.0$

the total energy dissipation for a microplane in the normal direction is obtained as by integrating the dissipation rates over the pseudo time variable t :

$$\mathcal{D}_{\text{int}}^N = \int_0^t [\sigma_N \dot{\epsilon}_N^p - Z_N \dot{z}_N - X_N \dot{\alpha}_N + Y_N \dot{\omega}_N] dt. \quad (7)$$

The energy dissipation for the tangential direction is evaluated in the same way as for the normal direction. Starting with the Clausius-Duhem inequality, i.e.

$$\dot{\mathcal{D}}_{\text{int}}^T = -\rho \dot{\psi}_T + \sigma_T \dot{\epsilon}_T \geq 0, \quad (8)$$

and the expression for the energy dissipation in terms of the state variables

$$\dot{\mathcal{D}}_{\text{int}}^T = - \left(\frac{\partial \rho \psi_T}{\partial \epsilon_T^e} \dot{\epsilon}_T^e - \frac{\partial \rho \psi_T}{\partial \epsilon_T^p} \dot{\epsilon}_T^p + \frac{\partial \rho \psi_T}{\partial z_T} \dot{z}_T + \frac{\partial \rho \psi_T}{\partial \alpha_T} \dot{\alpha}_T + \frac{\partial \rho \psi_T}{\partial \omega_T} \dot{\omega}_T \right) + \sigma_T \dot{\epsilon}_T. \quad (9)$$

the first term represents the tangential stress

$$\sigma_T = \frac{\partial \rho \psi_T}{\partial \epsilon_T^e}, \quad (10)$$

integrating over the loading history represented by the pseudo-time t , the total energy dissipation reads

$$\mathcal{D}_{\text{int}}^T = \int_0^t [\sigma_T \cdot \dot{\epsilon}_T^p - Z_T \dot{z}_T - X_T \cdot \dot{\alpha}_T + Y_T \dot{\omega}_T] dt. \quad (11)$$

The macroscopic energy dissipation is evaluated by integrating the contributions of each microplane for the normal and tangential directions:

$$\mathcal{D}_{\text{int}}^{\text{mac}} = \frac{3}{2\pi} \int_{\Omega} \mathcal{D}_{\text{int}}^{\text{mic}} d\Omega = \frac{3}{2\pi} \int_{\Omega} \mathcal{D}_{\text{int}}^N d\Omega + \frac{3}{2\pi} \int_{\Omega} \mathcal{D}_{\text{int}}^T d\Omega. \quad (12)$$

2.3 Elementary study at microplane level

To illustrate the possibility of evaluating the individual energy fractions, an elementary study of the cyclic behavior of a single microplane for the tangential direction is displayed in Fig. 1. The stress response to a displacement-controlled cyclic load is shown in Fig. 1a. Large damage and plastic values are reached during the first loading cycle, as shown in Fig. 1a by the loss of stiffness during the first unloading step and the amount of irreversible strain when passing through the zero stress level. Fig. 1c documents the ability of the model to slow down the damage accumulation for successive loading cycles, which represents an essential feature of models that can realistically capture the concrete fatigue response. Fig. 1b shows the numerically quantified stored and dissipated energy at the interface. It can be seen that the damage dissipation increases mainly during the first cycles and then the increment rate decreases, which is an obvious consequence of the above-mentioned slowdown of the damage evolution. As a consequence, the rate of plastic dissipation growth also slows down after a few cycles since the

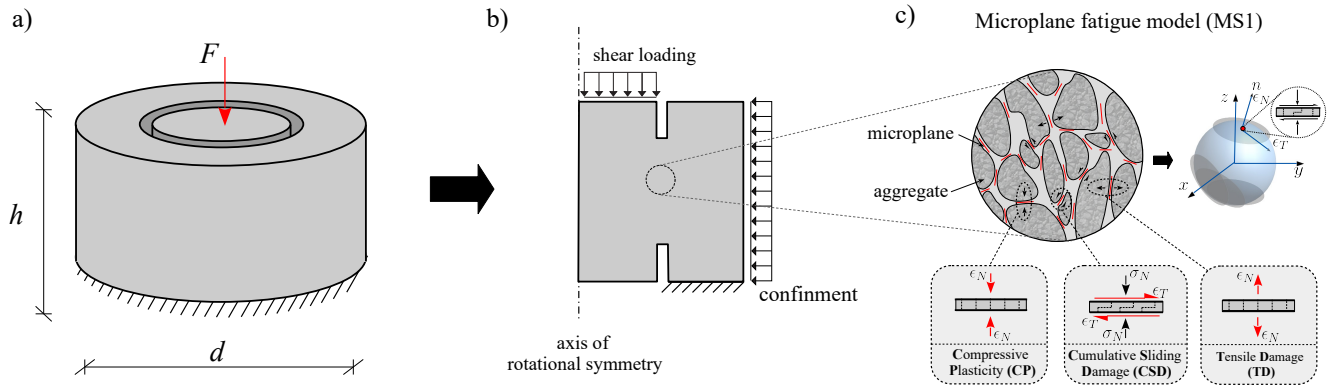


Fig. 2: a) Punch Through Shear Test (PTST); b) axisymmetric idealization of the PTST; c) concept of the used microplane fatigue model

growth of damage and plastic dissipation are explicitly linked. The isotropic free energy increases after a few cycles due to the progressive cumulative plastic process. The stored elastic energy shown in green increases during loading and decreases during unloading, reaching zero each time the stress-slip curve crosses the zero tangential stress axis. With each successive cycle, the available elastic strain decreases and, at the same time, the maximum tangential stress decreases, resulting in a decrease in the maximum stored elastic energy at the peak of each successive cycle.

3 Punch Thorough Shear Test (PTST)

The cylindrical punch through shear test (PTST) was originally introduced by Luong et al. [9] to investigate the material behavior of high-strength concrete under combined shear and compression load. Later, Backers et al. [10] used a similar test for measuring the rock fracture toughness in mode II. This test was recently used for preliminary studies of concrete fatigue to investigate the strong influence of compressive stress on fatigue life under fatigue load in Mode II [1]. It has been shown, that the test setup allowing for a simultaneous control of shear and compressive stresses can be used to systematically isolate the influence of the degree of confinement on the fatigue life of high-strength concrete under subcritical fatigue shear loading at [11, 12]. To enable an efficient cycle-by-cycle simulation of the fatigue response of PTST, a low-complexity axis-symmetric idealization shown in Fig. 2 is used here, that still realistically captures the governing principles of the fatigue development. This representation is valid if no radial splitting cracks occur. Since the numerical investigations will focus primarily on confined concrete under Mode II loading, the occurrence of splitting cracks can be ruled out, as documented for various types of rocks in [10]. The dimensions of the model correspond to a height h of 50 mm and a diameter d of 100 mm. The center of the notch of the ligament is situated 26 mm to the axis of symmetry and has a width of 4 mm.

3.1 Monotonic behavior

To quantitatively reproduce the experimental behavior of concrete, the material parameters of the MS1 model were calibrated using a uniaxial stress state to reproduce characteristic strength and stiffness values. The simulated compressive and tensile

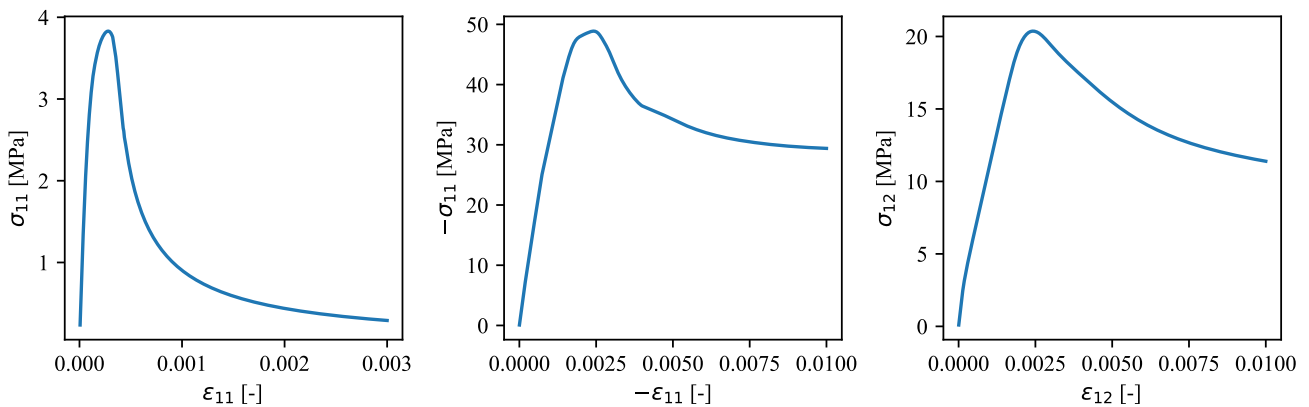


Fig. 3: Elementary tensile, compressive and shear strength. Material parameters: $E = 39000$, $\nu = 0.18$, $\epsilon_N^0 = 1e^{-5}$, $A_d = 7000$, $\sigma_N^0 = 25.0$, $\gamma_N = 8000$, $K_N = 14000$, $\sigma_T^0 = 2.2$, $\gamma_T = 65000$, $K_T = 12000$, $S_T = 0.0027$, $r_T = 9.2$, $c_T = 8.7$, $p_T = 7.5$, $m_T = 0.01$

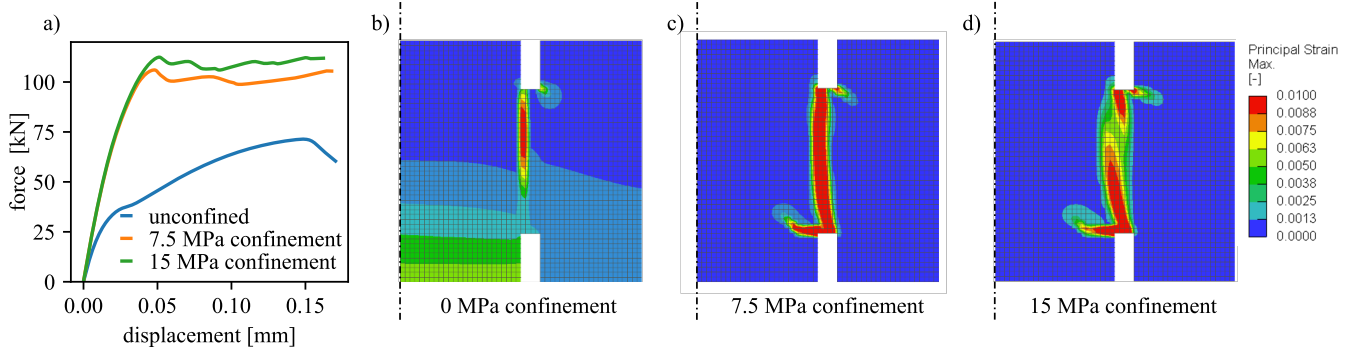


Fig. 4: a) force-displacement curve of the PTST for three considered ligament confinement levels, b), c) and d) ϵ_I for each considered confinement level at the end of the simulation

behavior, shown in Figs 3a, b demonstrates reasonable behavior for a normal-strength concrete with compressive and tensile strengths of 49 MPa and 3.7 MPa, respectively, and an E modulus of 39 GPa.

To investigate the ability of the MS1 microplane material model to reproduce the response of PTST under different normal confinement levels, Fig. 4 displays a study under mode II displacement control loading with the parameters calibrated from the studies under uniaxial loading. In particular, 3 levels of confinement were considered, which are 0, 7.5 and 15 MPa.

Fig. 4a displays the force-displacement curves for the three cases considered, where an increase in both peak load and stiffness is observed as the degree of confinement increases. It should be noted, that in case of the simulation with zero confinement, a strong deterioration in the circumferential direction develops, so that the validity of this result is limited by the occurrence of radial splitting cracks that violate the assumption of axi-symmetry.

As soon as the confinement reaches a certain level, the entire specimen is subjected to compressive stresses in the circumferential direction, so that the occurrence of splitting cracks is excluded. This is already the case at a confinement level of 7.5 MPa, as documented in Fig. 4c, where cracks propagate through the ligament, starting at the top and bottom of the notches until they coalesce and fracture the entire ligament.

A second set of radial Mode I cracks develops due to tensile stresses as a counterpart to the compression strut oblique to the vertical notch. This secondary set of cracks propagates only when the plateau on the force displacement curve is reached. The occurrence of such a crack type is documented in the literature by [10] and referred to as "wing cracks".

The specimen shown in 4d subjected to 15 MPa confinement shows a similar crack propagation pattern, except that the cracks propagating through the ligament propagate obliquely to the ligament and parallel to each other due to a higher tri-axial compression level in the ligament.

3.2 Cyclic behavior

To examine the ability of the model to reproduce the shear fatigue response under confinement, Fig. 5 presents a study under subcritical cyclic shear loading and confinement level of 7.5 MPa. The material parameters also correspond to the calibrated parameters from the studies under uniaxial loading depicted in Fig. 3.

In order to characterize the ongoing damage process that takes place at the ligament during the subcritical cyclic shear loading, and due to the computational costs, the MS1 microplane model is used only in the ligament of the specimen. This is justified by the fact that the failure of the specimen is due to crack propagation through the ligament. The rest of the specimen is modeled with a linear elastic material that has the same elastic modulus and Poisson's ratio as the ligament.

The first load range of the cyclic shear force was set between 70 kN for the upper level and 20 kN for the lower level. The force displacement curve and the breakdown of the dissipated energy are shown in Figs. 5a, b and c. It can be seen that the energy dissipation due to normal and tangential damage is of the same order of magnitude, while the dissipation due to tangential plasticity is two of order of magnitudes larger. Fig 5 d show the crack propagation pattern in terms of ϵ_I .

When the amplitude of the cyclic shear load is reduced to an upper value of 68 kN and a lower value of 20 kN, the fatigue life of the specimen is extended from 39 to 269 cycles. The associated force-displacement curve is shown in Fig. 5e and the corresponding energy dissipation in Figs. 5f and g shows that the energy dissipation due to tangential plasticity grows as the fatigue life of the material increases. On the other hand, the energy dissipation due to damage remains in the same order of magnitude. This result complies with the trends reported in [13], where the dissipation due to plasticity was increasing with the fatigue life of the specimen, while damage dissipation remained approximately constant.

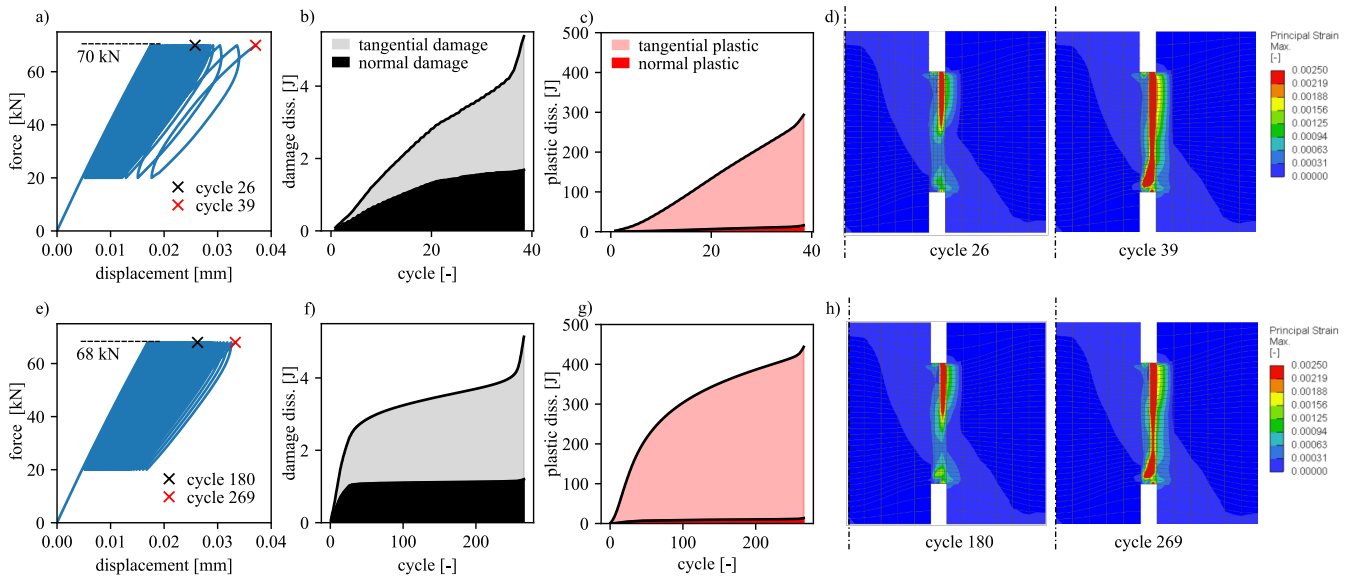


Fig. 5: a) and e) force-displacement curves for cyclic shear loading with range of 70 kN and 68 kN for the upper level and 20 kN for the lower level, respectively, b) and f) corresponding damage dissipation breakdown, c) and g) corresponding plastic dissipation breakdown, d) and h) ε_I at two different points of each respective simulation

4 Conclusions

To investigate the fatigue behavior of confined concrete subjected to fatigue shear loading, the microplane model MS1 was used in combination with a punch-through shear test (PTST) FE model. It was possible to simulate the damage propagation under confined pulsating subcritical shear loading, including a realistic reproduction of the shapes of the hysteretic loops. Moreover, the simulation with two different fatigue load amplitudes predicted larger number of cycles to failure for the lower amplitude, qualitatively complying to the Wöhler (S-N) curve of concrete. In addition, due to the pressure sensitivity included in MS1, several crack propagation patterns described in the literature for PTST under monotonic loading could be simulated. At the same time, the model could also reproduce the concrete response to elementary uni-axial monotonic loading using a single material point idealization. Summarizing, the studies covering a wide range of stress configurations, namely, monotonic uni-axial tension and compression, simple shear, confined shear, and fatigue confined shear, delivered plausible results using the same set of material parameters. Furthermore, based on the thermodynamic formulation of the MS1 microplane model, it was possible to evaluate the energy dissipation attributed to each dissipative mechanism. These studies reveal that plastic energy dissipation increases with fatigue life, while damage dissipation remains roughly constant for loading scenarios with different amplitudes. These insights can serve as a basis for more realistic predictions of fatigue life of concrete structures.

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